RADIATION FLAME FRONT IN A GAS MIXTURE WITH SOLID PARTICLES

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The problem about the stationary flame front structure in a gas suspension is solved on the basis of the representations, developed earlier, for the mechanics of multiphase multicomponent media in the presence of heterogeneous chemical reactions [1] within the framework of a diffusion (differential approximation) to describe the radiation field [2]. The behavior of the system of differential equations is investigated near the singularities corresponding to the initial and final states. Distributions of the parameters characterizing the gas, the particles, and the radiation field, as well as the dependences of the flame propagation velocity on a number of parameters governing the process under investigation (the particle diameter, mixture composition, etc.), are presented.

The particle surface temperature during their heterogeneous combustion can achieve such high values that the radiant heat transport becomes governing during the flame propagation over the suspension [3-5]. The gas, hence, has a lower temperature and is practically transparent in the wavelength band characteristic for the radiating particles.

Existing theories of the radiation flame front in gas suspension [3, 4] are based on the introduction of a mean radiation flux emerging from the high-temperature domain, and the existence of an ignition temperature is assumed. The gas mixture with the hot particles is, hence, considered approximately as a "gray" absorbing and emitting substance with the absorption coefficient $\varkappa = n\pi d^2/4$ [6], where n is the number of particles per unit volume, and d is the particle diameter.

Let us note [7] in which the problem of the radiation shock structure in a two-phase chemically inert medium is investigated in the diffusion approximation.

1. Basic Equation. Formulation of the Problem

The hydromechanics equations of a two-velocity, two-temperature continuous medium in the presence of heterogeneous chemical reactions have been obtained in [1] in application to a gas mixture with particles. For one-dimensional stationary motion taking account of heat transport by radiation, they are

$$\frac{dm_{11}}{dx} = -\nu_{11}J, \quad \frac{dm_{13}}{dx} = \nu_{13}J, \quad \frac{dm_{14}}{dx} = 0, \quad \frac{dm_2}{dx} = -\nu_2 J, \quad \frac{dnv_2}{dx} = 0$$

$$m_1\frac{dv_1}{dx} = -\alpha_1\frac{dp}{dx} - f + \nu_2 J(v_2 - v_1), \quad m_2\frac{dv_2}{dx} = -\alpha_2\frac{dp}{dx} + f$$

$$m_{11}\frac{d}{dx}\left(i_{11} + \frac{v_1^2}{2}\right) + m_2\frac{d}{dx}\left(i_2 + \frac{v_2^2}{2}\right) + m_{13}\frac{d}{dx}\left(i_{13} + \frac{v_1^2}{2}\right) +$$

$$+ m_{14}\frac{d}{dx}\left(i_{14} + \frac{v_1^2}{2}\right) = \frac{d}{dx}\lambda_1\frac{dT_1}{dx} - \frac{dq_R}{dx} + JQ^\circ + \nu_{11}J(c_{p1} - c_{p3})(T_1 - T_0) +$$

$$+ \nu_2 J\frac{p - p_0}{\rho_2^\circ} + \nu_2 J\left[c_2(T_2 - T_0) - c_{p3}(T_1 - T_0) + \frac{(v_2^2 - v_1^2)}{2}\right]$$

$$m_2\frac{du_2}{dx} = q_{12} - \frac{dq_R}{dx} + JQ^\circ + J[\nu_{11}(c_{p1}' - c_{p3}') + \nu_2(c_2 - c_{p3}')](T_2 - T_0)$$

$$\left(m_k = \rho_k v_1,\ m_2 = \rho_2 v_2,\ m_1 = \sum_k m_k,\ c_{p1}' = c_{p1}(T_2),\ c_{p3}' = c_{p3}(T_2)\right) \tag{1.1}$$

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Here a chemical reaction of the form $\varkappa_{11}A + \varkappa_2 B + \varkappa_{14} D = \varkappa_{13} C + \varkappa_{14} D$, is considered, where A, B, C, D are the symbols of chemical elements, and $\varkappa_k$ (k=11.2, 13, 14) are stoichiometric coefficients. Moreover, $\nu_k = g_k \varkappa_k$, where g_k are the molecular weights of the chemical elements, m_k are the mass flows of the components, α_1 and α_2 are the volume concentrations of the first and second phases, v_1 and v_2 are the phase velocities, p is the mixture pressure, u_k and $i_k = u_k + p/\rho_k{}^\circ$ are the internal energy and enthalpy, T_1 and T_2 are the phase temperatures, J is the chemical reaction rate per unit volume, f is the interaction force between the phases because of friction, q_{12} is the heat exchange between the phases, and q_R is the total radiation flux.

The parameters referring to the gas, the particles, and the radiation field will be provided with the subscripts 1, 2, and R everywhere below. The oxidizer, reaction products, and inert gas will be distinguished by using the second subscripts 1, 3, and 4, respectively. Longitudinal diffusion of the components is neglected here. Under the assumption of local thermodynamic equilibrium, the radiation transport equations in the diffusion approximation are [2]

$$\frac{dq_R}{dl_R} = c\left(\frac{4\sigma T_2{}^4}{c} - u_R\right), \qquad \frac{du_R}{dl_R} = -\frac{3}{c} q_R \qquad (1.2)$$
$$(dl_R = \varkappa dx,\ \varkappa = n\pi d^2/4)$$

Here u_R is the total radiation energy density, l_R is the optical thickness, σ is the Stefan–Boltzmann constant, and c is the velocity of light.

The first equation in (1.2) is the radiation continuity equation, and the second is the diffusion equation which establishes an approximate connection between the flux and the density of the radiation. This equation is valid in the case of weak anisotropy of the radiation field. Use of the diffusion approximation does not assume the radiation density to be equal to its equilibrium value.

The mass, force and thermal interaction between the phases is determined exactly as in [1]:

$$\begin{aligned}
J &= n\pi d^2 \frac{m_{11}}{\alpha_1 v_1} \frac{1}{1/\beta + 1/k} & &(\beta = N_{\mathrm{NU2}} D_{11}/d,\ k = z\exp(-E/RT_2)) \\
f &= n\frac{\pi d^2}{4} C_d \frac{m_1}{\alpha_1 v_1} \frac{(v_1 - v_2)^2}{2} & &\left(C_d = \frac{A}{N_{\mathrm{Re}}},\ N_{\mathrm{Re}} = \frac{\rho_1{}^\circ (v_1 - v_2) d}{\mu_1}\right) \qquad (1.3) \\
q_{12} &= n\pi d^2 h (T_1 - T_2) & &\left(h = \frac{\lambda_1 N_{\mathrm{Nu1}}}{d},\ N_{\mathrm{Nui}} = N_{\mathrm{Nui}}(N_{\mathrm{Re}})\right)
\end{aligned}$$

Here β and h are the heat and mass emission between the particles and the gas, D_{11} is the self-diffusion coefficient of the oxidizer, k is a chemical reaction rate constant, E is the activation energy, z is the pre-exponential factor, λ_1 and μ_1 are the heat conductivity and viscosity coefficients of the gas phase, C_d is the friction coefficient, A is a numerical coefficient (in particular, A=24 for Stokes law), and N_{Re}, N_{Nu1}, N_{Nu2} are the Reynolds, thermal, and diffusion Nusselt numbers, respectively.

Let us assume that the components of the first and second phases satisfy the equations of state

$$\begin{aligned}
i_{11} &= c_{p1}(T_1 - T_0) + h_{11}{}^\circ & &(p_{11} = \rho_{11}{}^\circ R_{11} T_1) \\
i_2 &= c_2(T_2 - T_0) + h_2{}^\circ + (p - p_0)/\rho_2{}^\circ & &(\rho_2{}^\circ = \mathrm{const}) \\
i_{13} &= c_{p3}(T_1 - T_0) + h_{13}{}^\circ & &(p_{13} = \rho_{13}{}^\circ R_{13} T_1) \\
i_{14} &= c_{p4}(T_1 - T_0) + h_{14}{}^\circ & &\left(p_{14} = \rho_{14}{}^\circ R_{14} T_1,\ p = \sum_k p_k,\ h_{14} = 0\right) \qquad (1.4)
\end{aligned}$$

Here h_i (i=11, 13, 14, 2) is the enthalpy of the components for $T = T_0$, $p = p_0$.

Equations (1.1) have the six first integrals

$$\begin{gathered}
m_1 + m_2 = \mathrm{const},\ m_{14} = \mathrm{const},\ m_{11} - \nu_{11} \cdot \nu_2{}^{-1} m_2 = \mathrm{const} \\
n v_2 = \mathrm{const},\ m_1 v_1 + m_2 v_2 + p = \mathrm{const} \\
(n = 6\alpha_2/\pi d^3) \\
m_{11}\left(i_{11} + \frac{v_1{}^2}{2}\right) + m_2\left(i_2 + \frac{v_2{}^2}{2}\right) + m_{13}\left(i_{13} + \frac{v_1{}^2}{2}\right) + m_{14}\left(i_{14} + \frac{v_1{}^2}{2}\right) \\
-\lambda_1 \frac{dT_1}{dx} + q_R = \mathrm{const} \qquad (1.5)
\end{gathered}$$

For optically thick flame fronts whose dimension l is very much greater than the radiation mean free path l_2 ($l_2/l \ll 1$), the radiation flux from the high temperature domain will equal in order of magnitude

$$q_R \sim \sigma T_2^4\, l_2 / l \tag{1.6}$$

in conformity with (1.2).

Then the interrelationship between the heat flux, because of heat conduction over the gas and because of radiation, is defined by the dimensionless parameter

$$\Pi = \sigma T_2^3 l_2 / \lambda_1 \tag{1.7}$$

If the particle temperature reaches high values during combustion so that $\Pi \gg 1$, then the term taking account of heat transmission because of heat conduction in (1.1) is negligible. In this case, the flame propagation over the mixture occurs only because of radiation heat transfer.

The boundary conditions of the problem of steady flame front propagation are values of the parameters characterizing the state of the system before $(x = -\infty)$ and after $(x = +\infty)$ the flame front, which will be denoted by (0) and (e), respectively.

Let us assume the states (0) and (e) to be characterized by complete thermodynamic equilibrium. Then, it is necessary to give

$$\begin{gathered} p_0,\ v_0,\ T_0,\ m_{110},\ m_{20},\ m_{130},\ (dT_1/dx)_0 = (dT_2/dx)_0 = 0 \\ p_e,\ v_e,\ T_e,\ m_{11e},\ m_{2e},\ m_{13e},\ (dT_1/dx)_e = (dT_2/dx)_e = 0 \end{gathered} \tag{1.8}$$

respectively in the (0) and (e) states.

The natural boundary conditions for the radiation field parameters in the (0) state are

$$u_{R0} = 4\,\sigma T_0^4 / c, \qquad q_{R0} = 0 \tag{1.9}$$

When particles are completely absent in the (e) state ($m_{2e} = 0$), it is natural to give

$$u_{Re} = \text{const}, \qquad q_{Re} = \text{const} \tag{1.10}$$

In the case of a propellant excess, when $m_{2e} \neq 0$, it is necessary to give

$$u_{Re} = 4\,\sigma T_e^4 / c, \qquad q_{Re} = 0 \tag{1.11}$$

in the (e) state.

Let us note that the problem is completely defined by the boundary conditions given for the particle temperature when the radiation field is described by the exact radiation transport equation [8]. The use of the diffusion approximation simplifies the problem substantially but raises the order of the transport equation. Hence, in this case still another boundary condition must be given for one of the field parameters, for example, the first relationships (1.9), (1.10), and (1.11), where the second relationships are consequences of the first, and also (1.2) and the conditions (1.8).

The solution of the problem of steady flame front propagation reduces to seeking the integral curves of (1.1), (1.2) passing through the two singularities corresponding to the equilibrium states, i.e., to finding the eigenvalue which is the front propagation velocity in this case.

2. Investigation of the Behavior of the System near the Equilibrium States

The reaction rate equals zero ($J = 0$) near the initial state, and hence,

$$m_{11} = m_{110},\ m_2 = m_{20},\ \varkappa = \varkappa_0$$

Moreover, in the low-temperature region the radiation density is very much greater than the equilibrium value ($u_R \gg 4\,\sigma T_0^4/c$). Then (1.2) have the solution

$$q_R = q_{Rb} \exp(\sqrt{3}\, l_R), \qquad u_R = -\sqrt{3}\, q_{Rb} c^{-1} \exp(\sqrt{3} l_R) \tag{2.1}$$

The quantity q_{Rb} corresponds to a sufficiently small deviation of the radiation flux from the initial zero value. It is assumed that the solution of the exact nonlinear system should already be used when this flux is exceeded.

Using the equations of state (1.4) and the integrals (1.5), we obtain a solution of (1.1) in the form

$$\frac{T_1 - T_0}{T_{1b} - T_0} = \exp(\sqrt{3}\, l_R), \qquad \frac{T_2 - T_0}{T_{2b} - T_0} = \exp(\sqrt{3}\, l_R)$$

$$v_1 - v_0 = \frac{(m_{110} R_{11} + m_{14} R_{14})(T_{1b} - T_0)}{\alpha_0 p_0} \exp(\sqrt{3}\, l_R)$$

$$v_2 - v_0 = \frac{\varepsilon_1 (m_{110} R_{11} + m_{14} R_{14})(T_{1b} - T_0)}{\alpha_0 p_0} \exp(\sqrt{3}\, l_R)$$

$$\left(\varepsilon_1 = \frac{\pi n_0 d_0 A \mu_1}{\pi n_0 d_0 \mu_1 A + 16 \sqrt{3}\, m_{20} \varkappa_0}\right) \tag{2.2}$$

The relationships

$$T_{1b} = T_0 - \varepsilon_2 q_{Rb} / m_{20} c_2, \quad T_{2b} = T_0 - \varepsilon_3 q_{Rb} / m_{20} c_2 \tag{2.3}$$

$$\varepsilon_2 = \frac{2\pi n_0 d_0 \lambda_1}{\sqrt{3}\, \varkappa_0 (m_{110} c_{p1} + m_{14} c_{p4}) + 2\pi n_0 d_0 \lambda_1 [1 + (m_{110} c_{p1} + m_{14} c_{p4}) / m_{20} c_2]}$$

$$\varepsilon_3 = \frac{2\pi n_0 d_0 \lambda_1}{\sqrt{3}\, \varkappa_0 m_{20} c_2 + 2\pi n_0 d_0 \lambda_1 (1 + m_{20} c_2 / (m_{110} c_{p1} + m_{14} c_{p4}))}$$

should hence be satisfied.

Therefore, just one of the parameters T_{1b}, T_{2b}, q_{Rb} is independent.

Let us turn to an investigation of the behavior of the medium near the (e) state. This state is characterized by the asymptotic tendency of the reaction rate to zero as $m_{11} \to m_{11e}$, $m_2 \to m_{2e}$. Let us assume the parameters of the first phase and the velocity of the second to be already the equilibrium values [1]

$$v_1 = v_2 = v_e, \quad p = p_e, \quad T_1 = T_e, \quad N_{\mathrm{Nu}1} = N_{\mathrm{Nu}2} = 2 \tag{2.4}$$

Using the asymptotic expressions for the reaction rate in the diffusion combustion mode [1] for stoichiometric mixture, excess oxidation, and excess propellant cases, respectively, we obtain asymptotic expressions for the optical thickness

$$l_{Re} - l_R = \frac{3\zeta}{\nu_2 \eta D_{11}} m_2^{1/3}, \qquad l_{Re} - l_R = \frac{3\zeta}{4 \nu_2 \eta D_{11} m_{11e}} m_2^{4/3}$$

$$l_R - l_R{}^\circ = \frac{\zeta m_{2e}{}^{1/3}}{\nu_{11} D_{11} \eta} \ln m_2{}'^\circ / m_2{}' \tag{2.5}$$

$$\left(m_2{}' = m_2 - m_{2e}, \ \eta = \frac{12 m_{20}{}^{2/3}}{\rho_2{}^\circ d_0{}^2 \alpha_0 v_e}, \ \zeta = \frac{6 m_{20}{}^{1/3}}{\rho_2{}^\circ d_0 \alpha_0}\right)$$

where the zero superscript denotes parameters corresponding to some fixed state from which the asymptotic solution starts to be used.

Let us examine the stoichiometric mixture. Using the energy equation of the second phase [the last equation in (1.1)] and the relationship (1.3), we obtain an equation to describe the system behavior near the singularities in the variables (T'_2, m_2):

$$\frac{dT_2{}'}{dm_2} = \frac{a_1 T_2{}' - b_1 m_2 + e_1 m_2{}^{1/3} \varphi_R}{m_2{}^2} \qquad \left(\varphi_R = \frac{dq_R}{dl_R}, \ T_2{}' = T_2 - T_e\right) \tag{2.6}$$

$$\left(a_1 = \frac{12 m_{20}{}^{2/3} \lambda_1}{\alpha_0 \rho_2{}^\circ d_0{}^2 \eta c_2 \nu_2 D_{11}}, \ b_1 = \frac{Q^\circ - (T'_e - T_0)[\nu_{11}(c_{p1}{}' - c_{p3}{}') + \nu_2 (c_2 - c_{p3}{}')]}{\nu_2 c_2}, e_1 = \frac{\zeta}{\nu_2 \eta D_{11}}\right)$$

It can be seen by direct integration of (2.6) that if the function φ_R HE tends to zero as $m_2 \to 0$ or tends to zero more slowly than $m_2{}^{2/3}$, then the derivative dT_2/dm_2 is negative, which contradicts the physical meaning. By using (2.5) it can be shown that the solution of (1.2), (2.6) which satisfies the condition $dT_2/dm_2 > 0$ as $m_2 \to 0$ is to the higher-order accuracy

$$T_2{}' = \frac{b_1}{3 a_1 e_1} (l_{Re} - l_R)^3, \qquad q_R = -\frac{\varepsilon_4}{12} (l_{Re} - l_R)^4 \tag{2.7}$$

$$\varphi_R = \frac{\varepsilon_4}{3} (l_{Re} - l_R)^3 \qquad \left(\varepsilon_4 = \frac{16 \sigma T'_e{}^3 b_1}{a_1 e_1}\right)$$

If there is an excess of oxidizer in the mixture, then the equation in the (T'_2, m_2) variable is

$$\frac{dT_2'}{dm_2} = \frac{a_2 T_2' - b_2 m_2 + e_2 \varphi_R m_2^{1/3}}{m_2}$$

$$\left(a_2 = a_1 / m_{11e} - (\nu_{11}(c_{p1}' - c_{p3}') + \nu_2 (c_2 - c_{p3}')) / \nu_2 c_2,\ T_2' = T_2 - T_{2e}\right.$$
$$\left. b_2 = \frac{a_1}{\nu_2},\ T_{2e} = \frac{T_e a_1 / m_{11e} + Q^\circ + T_0 (a_1 / m_{11e} - a_2)}{a_2},\ e_2 = e_1 \nu_{11} / m_{11e};\ a_2 \gg 1\right) \tag{2.8}$$

Exactly as in the previous case, it can be shown that the solution of (2.8), (1.2) which satisfies the condition of decreasing temperature T_2' as $m_2 \to 0$ is to higher-order accuracy

$$T_2 = \frac{4b_2}{3(a_2 - 1)e_2}(l_{Re} - l_R)^{3/4}, \qquad q_R = q_{Re} - \frac{16\varepsilon_5 (l_{Re} - l_R)^{7/4}}{21}$$
$$\varphi_R = \frac{4\varepsilon_5}{3}(l_{Re} - l_R)^{3/4} \qquad \left(\varepsilon_5 = 16\sigma T_{2e}^3 \frac{b_2}{(a_2 - 1) e_2}\right) \tag{2.9}$$

In the case of a propellant excess, asymptotic solutions of the equations are easily obtained; hence, in the limit as $m_2' \to 0$ we have $T_e \to T_e'$, $q_R \to 0$ and $\varphi_R \to 0$.

3. Analysis of the Radiation Flame Front Structure in a Gas Suspension

For convenience in carrying out the numerical computations, the system (1.1), (1.2) is reduced to a dimensionless form solved for the highest derivatives [1]. Selected as dimensionless variables are

$$P = \frac{p}{p_0},\ U_i = \frac{v_i}{a_0},\ \theta_i = \frac{T_i}{T_0},\ M_k = \frac{m_k}{m_{10}},\ q_R^* = \frac{q_R}{\rho_{10} a_0^3}$$
$$u_R^* = \frac{u_R c}{\rho_{10} a_0^3} \quad \left(a_0^2 = \frac{\gamma_{10} p_0}{\rho_{10}};\ i = 1, 2;\ k = 1, 11, 13, 14, 2\right) \tag{3.1}$$

In order to find the integral curve passing through the two singularities corresponding to the system state ahead of ($x = -\infty$) and behind ($x = \infty$ or $x = x_\delta$) the front numerically, an adjustment must be made according to the parameter U_0. For $x = 0$, let us determine a θ_{2b} such that $1 < \theta_{2b} < \theta_e$, where θ_{2b} must be taken sufficiently close to unity. The quantities θ_{1b}, q_{Rb} are determined according to θ_{2b} in conformity with the asymptotic solutions (2.1)-(2.3). Furthermore, let us select U_0 so that the values of the parameters in the limit ($x \to \infty$ or $x \to x_\delta$) would yield values corresponding to the state (e) upon integration of the system to the right from $x = 0$. The asymptotic behavior of the parameters near the state (e) is determined in conformity with the solutions (2.7), (2.9).

4. Results of the Computations

As an illustration, the combustion of carbon particles in air is considered. The papers [3, 9] are devoted to an experimental investigation of flame propagation in a carbon–air mixture. The relatively low values of the flame propagation velocity (~20 cm/sec) obtained by using a burner [9] were the reason why the combustion of such mixtures was examined from the thermal theory viewpoint in a number of theoretical papers [1, 10]. However, the clearly exaggerated values of the particle temperature in the computations [1] indicate that radiation must be taken into account in investigations of carbon–air flames. The experiments [3, 4] also indicate this (particles of polymer materials were used in [4]), where the flame propagation in pipes was investigated. High values of the velocity (~1-5 m/sec) and the front length (1-5 m) characteristic for the radiation mechanism of flame propagation are obtained in these papers.

Carbon combustion is accompanied by different chemical reactions which generally result in the formation of carbon dioxide in the long run. This circumstance is particularly essential for mixtures with a large excess of propellant when the reaction of reducing carbonic acid, which proceeds at high temperatures, plays an important part. Because of the reaction of total combustion of carbon dioxide at not too high temperature ($T < 1800$°K), the combustion of the carbon particles occurs as though only carbonic acid had been formed [11]. In this case, the heterogeneous reaction of carbon combustion according to the equation $O_2 + C = CO_2$ can be computed approximately.

Figures 1-3 present the results of numerical integration representing the structure of the radiation flame front ($v_0 = 23.5$ m/sec) in a gas suspension with the initial composition $M_{20} = 0.05$ (an excess of oxidizer), the initial particle diameter $d_0 = 50\ \mu$, and the following thermodynamic data:

$$p_0 = 1, \quad T_0 = 300, \quad \gamma_{10} = 1.41, \quad \rho_{10}^\circ = 0.118, \quad \rho_2^\circ = 2.2$$
$$c_{p1} = 0.915, \quad c_2 = 0.714, \quad c_{p3} = 0.84, \quad c_{p4} = 1.1$$
$$\lambda_{11}^\circ = 5.89 \cdot 10^{-5}, \quad \lambda_{13}^\circ = 3.28 \cdot 10^{-5}, \quad \lambda_{14} = 6.6 \cdot 10^{-5}$$

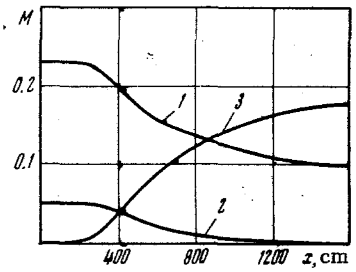

Fig. 1

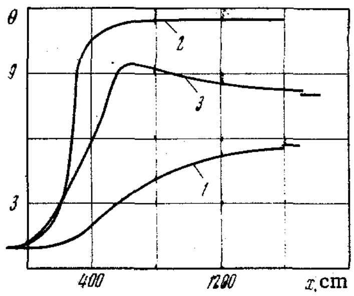

Fig. 2

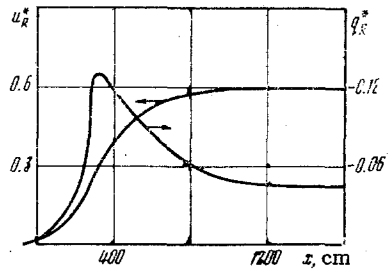

Fig. 3

$$D_{110} = 0.186, \quad \mu_{110} = 202 \cdot 10^{-6}, \quad \mu_{130} = 146 \cdot 10^{-6}$$

$$\mu_{140} = 182 \cdot 10^{-6}, \quad Q^{\circ} = 94052,$$

Here p is in atm, T_0 in °K, ρ in g/cm^3, c in J/g·deg, λ in cal/cm·sec·deg, D in cm^2/sec, μ in g/cm·sec, and Q° in cal.

The temperature dependences of the thermodynamic coefficients (c_p, λ, D, μ) were taken in conformity with [12]. The kinetic constants were taken from [11] to be $E = 4 \cdot 10^4$ cal/mole and $z = 5 \cdot 10^8$ cm/sec.

The following dependences were assumed for the dimensionless coefficients governing the interphasal interaction [11, 13].

$$C_d = \frac{52}{N_{\mathrm{Re}}}\left(\frac{\theta_2}{\theta_1}\right)^m, \quad N_{\mathrm{Nu}1} = \frac{0.56 N_{\mathrm{Re}}^{1/2}}{1 - \exp(-0.28 N_{\mathrm{Re}}^{1/2})}, \quad N_{\mathrm{Nu}2} = \frac{0.7 N_{\mathrm{Re}}^{1/2}}{1 - \exp(-0.35 N_{\mathrm{Re}}^{1/2})}$$

Here m is the exponent characterizing the temperature dependence of the kinematic viscosity.

It follows from the graphs represented that gradual heating of the particles occurs first (curve 2 in Fig. 2) because of particle absorption of the radiation issuing from the high-temperature domain. The reaction rate is low in this domain. Later the reaction rate starts to rise noticeably and to become so great that the heat being liberated in the particles does not succeed to be removed to the gas, and the particles start to be heated up spontaneously. This heating is particle deflagration; hence, the passage to the diffusion combustion domain occurs. The reaction rate in this domain depends slightly on the temperature; hence, growth in the particle temperature is retarded. Because of the diminution (Fig. 1) in the available carbon (curve 2) and oxygen (curve 1) masses, diminution in the reaction rate occurs (diminution in the heat evolution in the particles), and the particle temperature drops gradually because of heat losses by radiation and heat supply to the gas, which tends to its equilibrium value as the particles burn up.

The radiation energy density (Fig. 3) is greater than the equilibrium value in the preheating zone and less in the combustion zone. The inflection in the curve u_R^* (x) corresponding to the maximum value of the radiation flux q_R^* separates the flame front into two domains. Ahead of the inflection point the particles absorb more energy than they radiate ($dq_R^*/dx < 0$) and are heated by radiation; after the inflection the particles emit more energy than they absorb ($dq_R^*/dx > 0$) and are cooled by radiation. As the particles burn up, the radiation density tends to the equilibrium value corresponding to the particle equilibrium temperature θ_{2e}, and the total radiation flux takes on a constant value different from zero. An increase in the gas temperature (curve 1 in Fig. 2) occurs because of its being heated by hotter particles. The gas temperature is, hence, practically unchanged in the preliminary heating zone. This is explained by the fact that the flame front is propagated so rapidly in this case that the particles are ignited more rapidly than they succeed in giving off part of their energy to the colder gas. The gas is heated mainly in the diffusion combustion zone; hence, its temperature reaches the equilibrium value determined by the heat of chemical reaction.

As computations by means of thermal theory have shown [1], the gas velocity because of thermal expansion increases sufficiently sharply, but the more inert particles are entrained in the gas motion with a certain delay; hence, the originating flow around the particles substantially influences the velocity v_0. Thermal gas expansion occurs more slowly when radiation is taken into account, and the particles are entrained at once in the gas motion. Computations taking account of the possibility of relative motion of the phases show that the particle velocity in this case agrees with the gas velocity on the whole extent of the front.

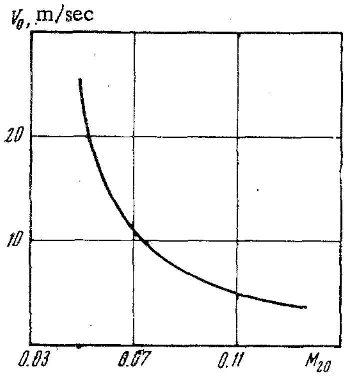

Fig. 4

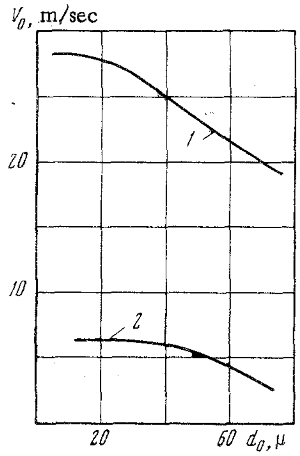

Fig. 5

Shown in Fig. 4 is the influence of the composition M_{20} of the fresh mixture (the remaining parameters are the same as indicated at the beginning of this section) on the flame propagation velocity therein. The diminution in the velocity v_0 as M_{20} increases is explained by the fact that at high flame propagation rates the radiation heat flux heats only the particles. The fact that v_0 diminishes as M_{20} increases has been noted in [4, 5].

The dependence of the flame front propagation velocity on the initial particle diameter d_0 is shown in Fig. 5 for a fixed fresh mixture composition (curve 1 for $M_{20} = 0.05$ and curve 2 for $M_{20} = 0.11$).

For small particle diameters, when combustion occurs in the kinetic domain,the time of combustion of a fixed mass of propellant increases as the particle diameter increases, but the radiation path length also increases, so that the velocity v_0 remains practically constant. For large particle diameters (the diffusion domain), the velocity v_0 diminishes as the diameter grows ($v_0 \sim d_0^{-m}$, $m \leqslant 5$).

Other conditions being equal, change in the kinetic constants can result in essential changes in the flow mode and the flame propagation velocity. The temperature distribution in the flame front ($v_0 = 89$ cm/sec) is shown in Fig. 2 (curve 3) for a stoichiometric mixture with $z = 5 \cdot 10^6$ cm/sec and $E = 4 \cdot 10^4$ cal/mole. In this case the flame is propagated so slowly that the gas and particle temperatures are practically the same on the whole extent of the front.

A deduction about their qualitative agreement can be made from a comparison between velocity values v_0 obtained and the results of an elementary analysis [3, 4]. However, the results of the elementary analysis depend strongly on the selection of the ignition temperature and the effective temperature of the radiating particles. The approach elucidated herein affords the possibility of taking account of chemical kinetics, heat transfer by radiation, and interphase interaction.

A comparison between experimental results [3] and results herein indicates their qualitative agreement.

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